

**“The Surface Model”
An Uncertain Continuous Representation
of the Generic Camera Model
and its Calibration**

Von der
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Dennis Rosebrock

Abstract

Using digital cameras for measurement purposes requires the knowledge of the mapping between 3D world points and 2D positions on the image plane. There are many different mathematical models that provide this mapping for a specific imaging system. These models tend to make assumptions about the structure of the system, e.g. an exact alignment of vision sensor and lenses or mirrors. If these constraints are not met or an inappropriate model is chosen, the measurement process will eventually deliver inaccurate results.

To avoid these problems, Grossberg and Nayar proposed a discrete generic camera model that describes a digital camera by assigning an arbitrary viewing ray to each pixel of the camera image. This makes the model applicable to any kind of camera, especially also to non-central ones like omnidirectional catadioptrics. However, this model is difficult to use in practice, as there is no direct method for mapping a 3D point to the image or determining rays for subpixel image positions.

In this work, the *Surface Model*, an uncertain continuous representation of the generic camera model, will be introduced. It uses a spline surface in 6D Plücker space to describe the camera. The interpolation abilities of the spline surface allow the viewing ray and its uncertainty for any (subpixel) position to be easily determined. Furthermore, the description facilitates the mapping from 3D world points to the image.

The calibration of the generic model has to be performed pixel-wise and is technically involved and time-consuming. In this work, hand-held sparse planar chessboard patterns are used for calibration. This introduces the assumption of a certain mapping continuity, but the calibration is much simpler to execute from a technical point of view. Furthermore, the uncertainties of the corresponding image point measurements are taken into account and propagated during the complete calibration procedure to obtain an uncertain model. It delivers uncertainty information in the form of covariance matrices for each camera operation. Simulations prove the validity of each step and the practical applicability of the procedure is shown by calibrating several real cameras of different types.

Kurzfassung

Um digitale Kameras zu Vermessungszwecken einzusetzen muss der mathematische Zusammenhang zwischen 3D Weltpunkten und 2D Bildpunkten bekannt sein. Es existiert eine Vielzahl an mathematischen Modellen, welche diese Abbildung für spezifische Kamerasysteme beschreiben. Für deren Gültigkeit ist die Einhaltung der zugehörigen Randbedingungen, beispielsweise die hochgenaue Ausrichtung von Bildsensor, Linsen und Spiegeln, zwingend erforderlich. Andernfalls können stark fehlerhafte Messergebnisse die Folge sein. Um diese Problematik zu meiden, haben Grossberg und Nayar ein diskretes generisches Kameramodell vorgeschlagen. Dieses zeichnet sich dadurch aus, dass jedem einzelnen Pixel ein separater Sehstrahl zugeordnet wird. Somit kann jede erdenkliche Kamera beschrieben werden. Dies gilt auch, wenn kein punktförmiges optisches Zentrum existiert, wie es zum Beispiel bei omnidirektionalen catadioptrischen Systemen der Fall sein kann. Aufgrund der Diskretisierung kann allerdings nicht für jede beliebige Subpixel-Position ein Sehstrahl ermittelt werden kann. Auch die Projektion eines beliebigen 3D-Punktes ins Kamerabild ist nicht ohne Weiteres möglich.

In dieser Arbeit wird das *Surface Model* vorgestellt. Es dient als eine kontinuierliche Repräsentation des generischen Kameramodells, welche Modellunsicherheiten explizit berücksichtigt. Zur mathematischen Beschreibung wird eine Splineoberfläche im sechsdimensionalen Plücker-Raum genutzt. Deren Interpolationsfähigkeiten erlauben es, für jedwede Subpixel-Position direkt einen Sehstrahl zu ermitteln, sowie einen beliebigen 3D-Punkt unmittelbar ins Kamerabild zu projizieren.

Die Kalibrierung des diskreten generischen Modells erfordert die Bestimmung mehrerer Messpunkte für jeden einzelnen Pixel. Entsprechende Verfahren sind zeitaufwändig und technisch anspruchsvoll. Um den Prozess zu vereinfachen, werden in dieser Arbeit von Hand platzierte, planare Schachbrettmuster eingesetzt.

Während der Messdatengewinnung für die Kalibrierung treten unweigerlich Messungenauigkeiten auf. Beim hier vorgestellten Verfahren zur Parameterermittlung des Surface Models werden diese Unsicherheiten explizit zur Stabilisierung und Verbesserung der Genauigkeit genutzt. Dies resultiert in einem unsicheren Kameramodell, welches für die Anwendungen der Sehstrahlermittlung und der Punktprojektion Ergebnisunsicherheiten in Form von Kovarianzmatrizen zur Verfügung stellt.

Mittels Simulationen wird die Anwendbarkeit sämtlicher vorgestellter Verfahren validiert. Durch die Kalibrierung verschiedener realer Kameras wird darüber hinaus deren praktische Nutzbarkeit aufgezeigt.

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