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**Jingming Hou**

**Robust Numerical Methods for  
Shallow Water Flows and Advective Transport  
Simulation on Unstructured Grids**

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**Robust Numerical Methods for Shallow Water Flows and Advective Transport Simulation on Unstructured Grids**

Kumulative Dissertationsschrift von Jingming Hou  
Fakultät VI – Planen, Bauen, Umwelt  
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# Publications of cumulative doctoral thesis

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3. Hou, J., Liang, Q., Simons, F. & Hinkelmann, R. (2013), ‘A 2D well-balanced shallow flow model for unstructured grids with novel slope source treatment’, *Advances in Water Resources* **52**, 107–131 (Impact Factor: 2.873).
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3. Hou, J., Simons, F., Busse, T. & Hinkelmann, R. (2010), Transport simulation in shallow water flows using adaptive methods, *in* ‘Proceedings of the First IAHR European Congress’, Edinburgh, UK.

4. Simons, F., Busse, T., Hou, J., Notay, K. & Hinkelmann, R. (2011), A robust and efficient solver for the shallow water equations and its application to a complex natural hydrosystem, *in* 'Proceedings of the 34th IAHR Congress', Brisbane, Australia, pp. 4276–4283.

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# Abstract

The two-dimensional (2D) shallow water equations (SWEs) are extensively used for hydrodynamic simulations in hydraulic and environmental engineering. The transport process inside shallow water, such as the transport of contaminant and sediment, can be modeled by solving the transport equation numerically. When solving the advective transport equation and SWEs, second order numerical schemes are widely used to reduce numerical diffusion caused by first order schemes. However, numerical oscillations may be induced by second order schemes without proper limiters. Second order total variation diminishing based flux limiting schemes (TVD schemes) are able to get rid of such numerical oscillations. In this cumulative dissertation, second order TVD schemes derived on 1D grids are extended to 2D unstructured grids, within the framework of the cell-centered finite volume method, to comfort to complex geometry. Moreover, an efficient treatment for slope source terms of SWEs and a robust approach handling wetting and drying are devised. This dissertation is on the basis of four papers in peer reviewed international journals and four conference contributions.

To extend second order TVD schemes to 2D unstructured grids, three methods are developed step by step. The first method adopts the TVD schemes which take the variation of cell size into account, for unstructured grids. In the second method, the first one is improved by applying more dominant upwind information perpendicular to the considered face. As a result, both the accuracy and efficiency are higher than the first one. Since an approximation is used, the accuracy of the second method is affected. By extrapolating the values of variables at the midpoint of the considered face, the third TVD method can produce more accurate results than the first two methods. The amelioration of each method in simulating linear advection is illustrated by the test cases in the corresponding papers and by a new test case in this dissertation. In addition, the third method is also employed to solve the SWEs.

A new treatment for the slope source terms of the SWEs is devised for unstructured grids. This treatment together with the hydrostatic non-negative water depth reconstruction method and the HLLC approximate Riemann solver, constitute a well-balanced scheme, which satisfies the conservation property.

In the case with the occurrence of wet-dry fronts, the very small water depths near wet-dry fronts may lead to unphysical high velocities and in turn to negative water depths. To preserve numerical stability, a new adaptive approach is proposed, by means of switching to first order scheme in such sensitive areas.

In this dissertation, the third method extending TVD schemes to 2D unstructured grids incorporated with the well-balanced scheme and the adaptive approach are proposed finally to simulate shallow water flows and advective transport inside. This model is able to get rid of numerical oscillations, to preserve the C-property and mass conservation, to achieve good convergence to steady state, to capture discontinuous flows and to handle complex flows involving wetting and drying over uneven beds, on unstructured grids with poor connectivity, in an accurate, efficient and robust way. These capabilities are verified against analytical solutions, numerical results of alternative models and experimental and field data.



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# Kurzfassung

Im Rahmen von hydraulischen und umwelttechnischen Fragestellungen sind auf der Lösung der zweidimensionalen Flachwassergleichungen basierende, hydrodynamische Simulationen weit verbreitet. Transportprozesse innerhalb Flachwasserströmungen, z.B. der Transport von Schadstoffen oder Sediment, können mit Hilfe der numerischen Lösung der Transportgleichungen modelliert werden. Zur Reduktion der numerischen Diffusion, die insbesondere bei numerischen Lösungsverfahren mit einer Genauigkeit erster Ordnung auftreten, werden häufig Verfahren zweiter Ordnung für die Lösung der advektiven Transport- und der Flachwassergleichungen verwendet. Ohne geeignete Limitierungen neigen Verfahren zweiter Ordnung jedoch zu numerischen Oszillationen. Sogenannte Total Variation Diminishing (TVD) basierte Verfahren zur Limitierung der numerischen Flüsse vermeiden numerische Oszillationen.

In dieser kumulativen Dissertation werden die TVD-Verfahren, die ursprünglich für eindimensionale Gitter hergeleitet wurden, im Rahmen der zellzentrierten Finite-Volumen-Methode auf zweidimensionale unstrukturierte Netze erweitert, um Berechnungen für komplexe Geometrien zu erlauben. Außerdem wird eine effiziente Behandlung des Sohlgefälles der Flachwassergleichungen und ein robuster Ansatz für die Behandlung von Benetzen und Trockenfallen entwickelt. Diese Dissertation basiert auf vier begutachteten Artikeln in internationalen Fachzeitschriften und vier Konferenzbeiträgen.

Schrittweise wurden drei Methoden zur Erweiterung der TVD-Verfahren auf zweidimensionale unstrukturierte Gitter entwickelt. In der ersten Methode werden die TVD-Verfahren, die die Variation der Zellengrößen berücksichtigen, auf unstrukturierte Gitter angewendet. Darauf aufbauend wurde in der zweiten Methode das vorgestellte Verfahren verbessert, indem mehr Informationen aus der Strömungsrichtung senkrecht zur betrachteten Zellkante berücksichtigt wurden. Durch zusätzliche Extrapolation der Variablen an den Mittelpunkten der betrachteten Zellkante erzeugt die als dritte vorgestellte Methode noch genauere Ergebnisse als die ersten beiden. Die Verbesserung jeder Methode bezüglich der Simulation von linearer Advektion wird in den entsprechenden Artikeln durch Testszenarien aufgezeigt, und ein neues Testszenario wird in dieser Dissertation präsentiert. Außerdem wird die dritte Methode zusätzlich auf die Lösung der Flachwassergleichungen übertragen.

Für die Behandlung des Sohlgefälles der Flachwassergleichungen wurde ein neues Verfahren entwickelt. Zusammen mit der hydrostatischen, stets positiven

Rekonstruktion der Wassertiefe und dem approximativen HLLC-Riemann-Löser erhält man so ein massen- und impulsconservatives sowie balanciertes Verfahren.

Beim Auftreten von Trockenfallen und Benetzen können sehr kleine Wassertiefen an der Benetzungsfront zu unphysikalisch hohen Fließgeschwindigkeiten und negativen Wassertiefen führen. Um numerische Stabilität zu erhalten, wurde ein neuer adaptiver Ansatz vorgeschlagen. Dieser basiert auf der Idee, die Genauigkeit des Verfahrens in kritischen Bereichen auf eine Genauigkeit erster Ordnung zu reduzieren.

In dieser Dissertation wird die dritte Methode zur Erweiterung der TVD-Verfahren auf zweidimensionale unstrukturierte Gitter schließlich mit dem zuvor beschriebenen Verfahren zur Lösung der Flachwassergleichungen für die Simulation von Strömungs- und Transportprozessen kombiniert. Das entstandene Modell zeigt keine numerischen Oszillationen, ist massen- und impulsconservativ und zeigt ein gutes Konvergenzverhalten für stationäre und instationäre Berechnungen. Des Weiteren werden Diskontinuitäten in der Strömung, Trockenfallen und Benetzen auf unebener Sohle auf unstrukturierten Gittern genau, effizient und robust gelöst. Vergleiche mit analytischen Lösungen, Ergebnissen alternativer Modelle und Daten aus Experimenten und Feldmessungen bestätigen die genannten Fähigkeiten.

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# Nomenclature

## Abbreviations

1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
BAP	basis of the averaging procedure
CCFV	cell-centered finite volume
COD	chemical oxygen demand
C-property	conservation property
CVs	conservative variables
ENO	essentially non-oscillatory
FCT	flux corrected transport
FEM	finite element method
FO	first order scheme
FOU	first order upwind
FSC	free surface correction
FVM	finite volume method
HLLC	Harten, Lax and van Leer approximate Riemann solver with the contact wave restored
HMS	Hydroinformatics Modeling System
HR	high resolution
LCD	limited central difference
MLP	multi-dimensional limiting process
MUSCL	monotone upstream schemes for conservation law
PDE	partial differential equation
PVs	primitive variables
SGM	surface gradient methods
SO	second order schemes
SWEs	shallow water equations
TVD	total variation diminishing
VFRs	volume-free surface relationships
WBAP	weighted biased averaging procedure
WENO	weighted essentially non-oscillatory

## Terms with Latin letters

$c$	concentration of the tracer transported	$[\text{kg}/\text{m}^3]$
$C$	upwind node or the cell whose centroid is $C$	
$C_f$	bed roughness coefficient	$[-]$
$D$	downwind node or the cell whose centroid is $D$	
$f$	face under consideration	
$F$	Froude number	$[-]$
$g$	gravity	$[\text{m}/\text{s}^2]$
$h$	water depth	$[\text{m}]$
$i$	index of cell	
$I$	intersection point at the considered face	
$k$	index of the faces of a cell	
$l$	area of a face (length for two dimensions)	$[\text{m}^2]$
$L$	left-side of $f$ or the left cell or node distance	
$L_1$	$L_1$ error	
$M$	midpoint of the considered face	
$n$	Manning coefficient or time level	$[\text{s}/\text{m}^{\frac{1}{3}}]$ or $[\text{s}]$
$q_c$	sink or source term of tracer	
$q_x, q_y$	unit-width discharges in $x$ - and $y$ -directions	$[\text{m}^2/\text{s}]$
$r$	r - factor	$[-]$
$R$	right-side of $f$ or the right cell	
$t$	time	$[\text{s}]$
$u, v$	velocities in $x$ - and $y$ -directions	$[\text{m}/\text{s}]$
$U$	further upwind node	
$x, y$	components of Cartesian coordinate	$[\text{m}]$
$z_b$	bed elevation	$[\text{m}]$
$\mathbf{f}, \mathbf{g}$	flux vectors in $x$ - and $y$ -directions	
$\mathbf{F}$	flux vector	
$\mathbf{n}$	unit outward vector normal to boundary	
$\mathbf{q}$	flow variable vector consisting of $h, q_x, q_y$ and $ch$	
$\mathbf{r}_{LI}$	position vector	
$\mathbf{S}$	source vector including slope source term $\mathbf{S}_b$ and friction source term $\mathbf{S}_f$	
$\mathbf{v}$	velocity vector	

## Terms with Greek letters

$\Gamma$	boundary of the control volume	
$\Delta$	variable difference	
$\varepsilon_{wd}$	wet-dry tolerance or threshold	[m]
$\eta$	water level $\eta = h + z_b$	[m]
$\underline{\nu}_t$	turbulent diffusivity tensor	[m <sup>2</sup> /s]
$\nu_w$	turbulent viscosity	[m <sup>2</sup> /s]
$\rho$	density of water	[kg/m <sup>3</sup> ]
$\phi$	transported variable	
$\psi$	flux limiter function	
$\Omega$	control volume	
$\nabla$	gradient operator	