Challenges in Number Theory

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Preface

Besides the main disciplines of interest and research in number theory, there are many riddles, problems, and conjectures, which sometimes seem to be simple and are always a challenge. They can be solved by everybody, by students, by specialists with greatest effort, or by nobody until now. This kind of problems was f.e. very popular in the times of Cl. G. Bachet, Sieur de Méziriac (Problèmes plaisans et délectables qui se font par les nombres, Lyon 1612). While not only professional but also non-professional mathematicians are busy with these tasks, they are not at all only entertainment or exercises and have led to several discoveries. In many cases, you can use well-known methods, but also new ways for number theory may be found, which is their main contribution to the latter.

In this text, which uses partly former publications of the author, you can find in 10 chapters examples of games, "magic" squares, riddles, secrets, and paradoxes, a discussion of the Ulam spiral, the Collatz conjecture, Catalan's conjecture/ Mihăilescu's theorem, Fermat's Last Theorem, other famous problems, some conjectures of primes, infinity and geometry, Zenon's Paradoxon, the problem of squaring the circle, and the meaning of vector spaces. After the text, a choice of literature is given.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in July 2010 Uwe Kraeft

http://www.kannitverstan.net/ shows contents and recent corrections of these studies in number theory

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions)
$\forall$	for all
$\in$	is element of (is contained in)
$\mathbb{N}$	natural numbers 1, 2, 3, ...
$\mathbb{N}^0$	natural numbers with zero 0, 1, 2, 3, ...
P	primes 2, 3, 5, 7, 11, ...
$\mathbb{Z}$	integers ..., -3, -2, -1, 0, 1, 2, 3, ...
$\mathbb{Q}, \mathbb{Q}^+$	rational numbers, positive rational numbers
$\mathfrak{R}$, $\mathfrak{I}$	real, imaginary part of a complex number z=x+iy
p_I	prime I (4n+1)
p_{II}	prime II (4m-1)
A={a,b,c}	an example of a set A with elements a, b, and c
i,j,m,n, ...	in this text, in most cases natural numbers or integers
#f(a)	number of elements within a class f(a)
=	equal
$\equiv$	$a \equiv b \pmod{c} \Leftrightarrow a \equiv b_c \Leftrightarrow (a-b)/c \in \mathbb{Z}$ for $a,b \in \mathbb{Z}$, $c \in \mathbb{N}$
$\cong$	so near as you want
$\approx$	about
$\sim$	similar
$\sum_{\nu=0}^{\infty}$	(infinite) sum
(x,y)=d	gcd of x and y is d

Other special symbols are explained in the text.

gcd	greatest common divisor
f.e.	for example (e.g.)

The order of this sequence of texts on number theory is twofold. The order following the date of printing is given at the end of this book (p. 59). Another grouping is got by the colours of the covers after disciplines as follows:

arithmetic number theory:	light blue
sequences and series:	dark green
Diophantine Equations:	orange
algebraic number theory:	dark red
topological number theory:	purple
analytic number theory:	dark blue
statistical number theory:	light green
special numbers:	dark yellow
textbooks:	light yellow

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