

Berichte aus der Mathematik

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**Knowledge Description and
Galois Correspondence**

Practical Impact of a Structural Idea

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Preface

The questions followed up in this monograph extend the results described in my book “Semantic and Syntactic Levels of Mathematical Thinking and Learning”. While evaluating my data, it became apparent that the theory of “knowledge spaces” provides a useful tool for the description and analysis of various kinds of mathematical knowledge including forms of cognitive misconception.

While further disassembling my results I was naturally led to the concept of a “Galois connection” which – although not noted explicitly – was first used by Galois in the 19th century. The structures concerned are – besides their relevance in cognitive science – of interest in their own right; they give rise to mathematical questions relating to several mathematical domains including set theory, algebra, and logic.

Aware as I was of the distinctive interrelations between theoretical and practical issues, I decided to present my results in an integral form; the present account is devoted to purely mathematical questions as well as to applications in cognitive science.

Besides perspectives of application, a special aspect inspired me while writing this book: the obvious beauty and aesthetic coherence of mathematical theory. The concepts and results to be described exhibit a distinctive clarity and elegance which, in conjunction with unpretentious simplicity, display a reality of exceptional, unconventional strength. May this impression as well as an appropriate sensibilization for the given interdisciplinary perspective be forwarded to the reader.

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